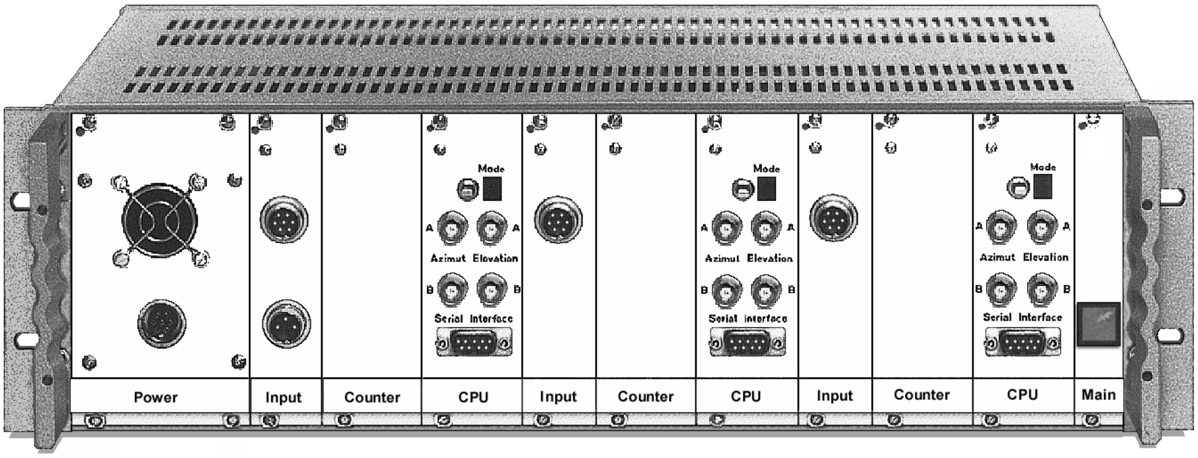
Angle-Meter

Measurement of 3D eye movements



Scleral search coil system for linear detection of three-dimensional angular movements

Primelec, D. Florin

Ostring 36
CH - 8105 Regensdorf
Switzerland

Phone: +41 1 884 28 84
FAX: +41 1 884 28 83

info@primelec.ch
www.primelec.ch

Introduction

Eye movements in 3D space are traditionally analyzed in terms of horizontal and vertical displacements of the line of sight and the rotation of the eye about the line of sight. Examples of these kinds of parameterizations of eye position are the coordinates introduced by Fick [1] and Helmholtz [2]. The output of the Angle-Meter is represented in a similar coordinate system. For describing the procedure of 3D eye (or head) movement recordings, we shall adopt a right-handed coordinate system and assume that the corresponding reference frames of the detector and the subject coincide in orientation.

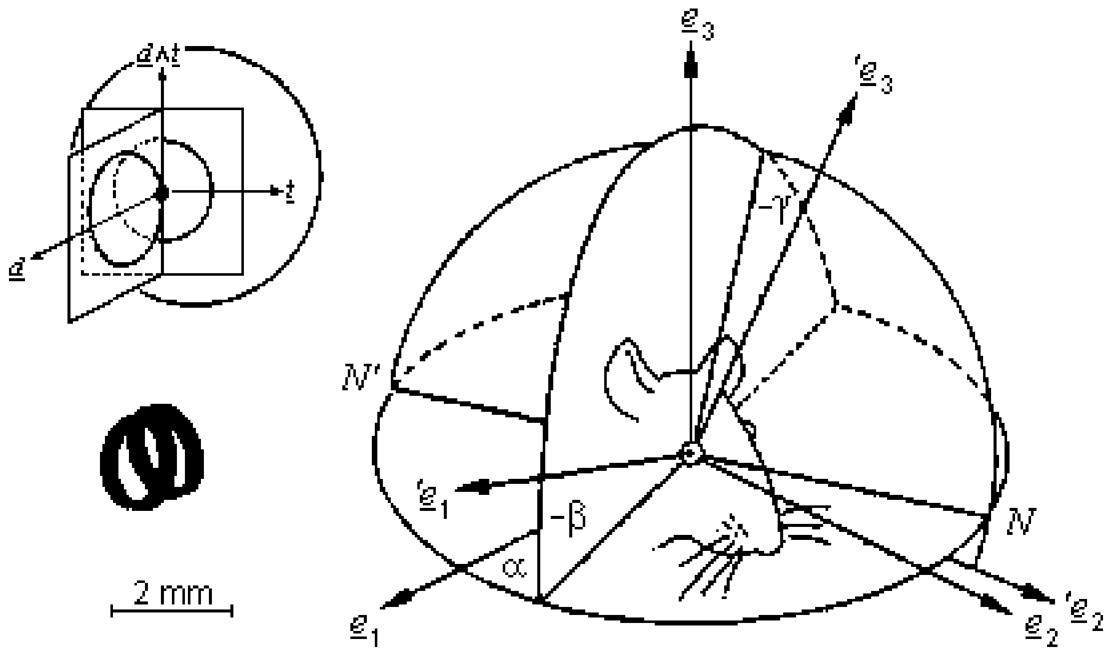


Figure 1. Principle of 3D eye movement measurements. The angular parameters α, β and γ describe the rotation of the eye from the initial position (reference position), defined by the orientation of the vectors $\underline{e}_1$, $\underline{e}_2$, and $\underline{e}_3$, to the final position indicated by the orientation of the vectors $'\underline{e}_1$, $'\underline{e}_2$, and $'\underline{e}_3$ (N' N is the nodal axis). The inset illustrates a miniature dual-search coil and, schematically, the orientation of such dual-search coil on the eyeball (direction coil vector $\underline{d}$, torsion coil vector $\underline{t}$, and cross product vector $\underline{d} \wedge \underline{t}$). Note that these vectors establish a reference frame rotating with the eyeball.

---

[1] A. Fick, "Die Bewegungen des menschlichen Augapfels," Z. rationelle Med., vol. 4, pp. 101-128. 1877.
[2] H. von Helmholtz, "Handbuch der Physiotogischen Oprik", Leipzig: Voss, 1986.

Technique

The direction of line of sight is determined by two angles (Fig. 1): a horizontal angle α defining how far the eye (line of sight) is rotated about a vertical axis $\underline{e}_3$ and a vertical angle β defining how far the eye is rotated about the nodal axis (axis $N' N$ in Fig. 1 defined by the intersection of the plane through the center of rotation perpendicular to the new direction of the line of sight and the horizontal plane). Both angles can be measured simultaneously by a search coil, which is attached to the eye concentrically to the pupil (so-called direction coil). The rotation of the eye about the line of sight (ocular torsion) defines a third angle γ as discussed below.

The outputs A (Azimuth) and B (Elevation) of the system are directly proportional to the angular displacements α and β of the search coil relative to the system's reference frame. The channel gains GA and GB are independent of the geometrical and electrical characteristics of an individual search coil. The orientation of the coil area vector $\underline{d}$ with respect to the system's frame of reference is given by the angular parameters α (= output A / channel gain GA) and β (= output B / channel gain GB) as follows:

$$\underline{d} = \cos\beta \cos\alpha \, \underline{e}_1 + \cos\beta \sin\alpha \, \underline{e}_2 - \sin\beta \, \underline{e}_3 \qquad (1)$$

Note that by the right-hand rule (i.e., downward movement positive), the component d_3 is negative for a positive angle β.

A complete description of the eye position in space requires recording of one more parameter that defines the state of rotation of the eyeball around the line of sight (ocular torsion). To this end, a second search coil (so-called torsion coil) must be attached to the eye in a non-parallel plane with respect to the first one (see the inset of Fig. 1). In such an arrangement, the outputs $A1$ and $B1$ (channel 1) and $A2$ and $B2$ (channel 2) define the angular orientation of coil 1 (i.e. direction coil with area vector $\underline{d}$) and coil 2 (i.e. torsion coil with area vector $\underline{t}$) in the system's reference frame. Given the fixed orientation of the two coils relative to each other, only three of the four parameters α_1 (= output $A1$ / channel gain $GA1$), β_1 (= output $B1$ / channel gain $GB1$), α_2 (= output $A2$ / channel gain $GA2$), and β_2 (= output $B2$ / channel gain $GB2$) are independent variables. The angle σ between the coil area vectors $\underline{d}$ (direction coil) and $\underline{t}$ (torsional coil) is determined by the scalar product (note that $\underline{d}$ and $\underline{t}$ are unity vectors):

$$\begin{aligned} \cos\sigma &= \underline{d} \bullet \underline{t} \\ &= \cos\beta_1 \cos\beta_2 \cos(\alpha_1 - \alpha_2) - \sin(\beta_1 - \beta_2) \end{aligned} \qquad (2)$$

The parameters α_1, α_2, β_1 and β_2 are related to the angle γ, which defines the amount of rotation (torsion) about an axis parallel to the direction coil axis as follows (for details, see 'Orthogonalization'):

$$\begin{aligned} &\mathrm{Cos}\,\gamma = \sin(\alpha_2 - \alpha_1) \cos\beta_2 / \sin\sigma \\ &\quad \gamma > 0 \ \text{ if } \ \sin\beta_2 > -\cos\sigma \sin\beta_1 \end{aligned} \qquad (3)$$

where σ is the angle between the direction and the torsion coil area vector. Given the angle σ, it is sufficient to measure the angles α_1, β_1 and β_2 to determine the parameters $\alpha = \alpha_1$, $\beta = \beta_1$, and γ, which provide a complete description of 3D eye positions.

Any eye position in head is given by a rotation $R = R(\underline{n}, \rho)$, which can be expressed as a rotation vector: $\underline{r} = \tan(\rho / 2)\, \underline{n}$, where $\underline{n}$ is the normalized axis about which the eye has to rotate in order to go from a reference position to the current position, and ρ is the angle of rotation about $\underline{n}$. The product of two rotations $R1R2$ has the same vector representation: $\underline{r}_1 \circ \underline{r}_2 = (\underline{r}_1 + \underline{r}_2 + \underline{r}_1 \wedge \underline{r}_2) / (1 - \underline{r}_1 \bullet \underline{r}_2)$, where $\circ$ indicates multiplication of rotation vectors, and ^ denotes the vector outher product [1], [2].

The rotation, bringing the eye from the reference position $\alpha = 0$, $\beta = 0$, and $\gamma = 0$ to the position defined by angles α, β and γ is given by the rotation vector:

$$\begin{aligned} \underline{r} = \{ &(\tan\gamma/2 - \tan\alpha/2 \tan\beta/2)\, \underline{e}_1 + (\tan\beta/2 + \tan\alpha/2 \tan\gamma/2)\, \underline{e}_2 \\ &+ (\tan\alpha/2 - \tan\beta/2 \tan\gamma/2)\, \underline{e}_3 \} \bullet 1 / (1 + \tan\alpha/2 \tan\beta/2 \tan\gamma/2) \end{aligned} \qquad (4)$$

---

[1] W. Haustein, "Considerations on Listing's law and the primary position ..." Biol. Cybern., vol. 60, pp. 411-420, 1989.
[2] K. Hepp, "On Listing's law," Comm. Math. Phys., vol. 132, pp. 285-292, 1990.

The orientation of the rotation axis is defined by $\underline{n} = \underline{r} / \|\underline{r}\|$ and the rotation angle by $\rho = 2 \tan^{-1} (\|\underline{r}\|)$ where $\| \ \|$ denotes the norm of a vector.

Examples of 3D eye movement records obtained from head-restrained rats with the Angle-Meter are illustrated in Figs. 2 and 3. The eye movements were recorded with a dual search coil consisting of two miniature coils (80 turns, diameter 2 mm, weight 1.1 mg) that were glued on top of each other in approximately perpendicular planes (see the inset of Fig. 1). The assembly was attached concentrically to the pupil onto the anaesthetized cornea of the right eye using a tiny drop of a histocompatible glue. The dual search coil was oriented on the eye such that one coil was plane on the cornea approximately aligned with the optic axis of the eye (direction coil) and the other coil approximately in a vertical plane (torsion coil). For further details, see [1].

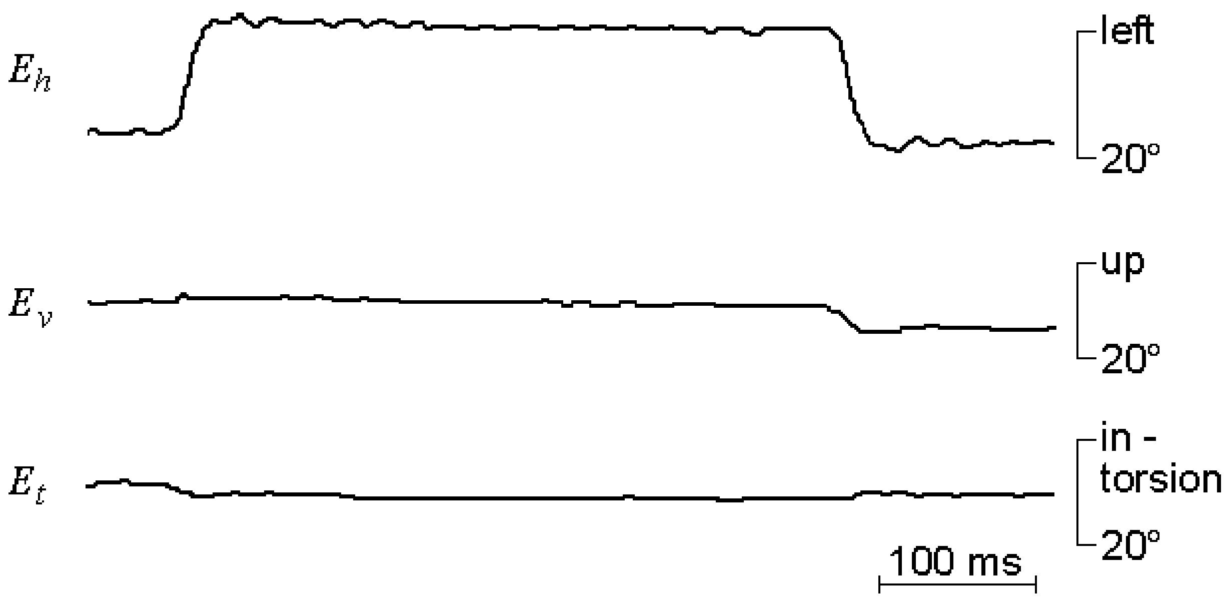


Figure 2. Example of saccades obtained in a head-restrained rat. The eye position traces E_h, E_v, and E_t correspond to the output signals $A1$, $A2$, and $B2$ of the Angle-Meter. No correction for true torsion nor for deviation from space quadrature of the dual-search coil (σ = 88°**).**

[1] B. J. M. Hess and N. Dieringer, "Spatial organization of the maculo-ocular ..." Euro. J. Neurosci., vol. 2, pp. 909-919, 1990.

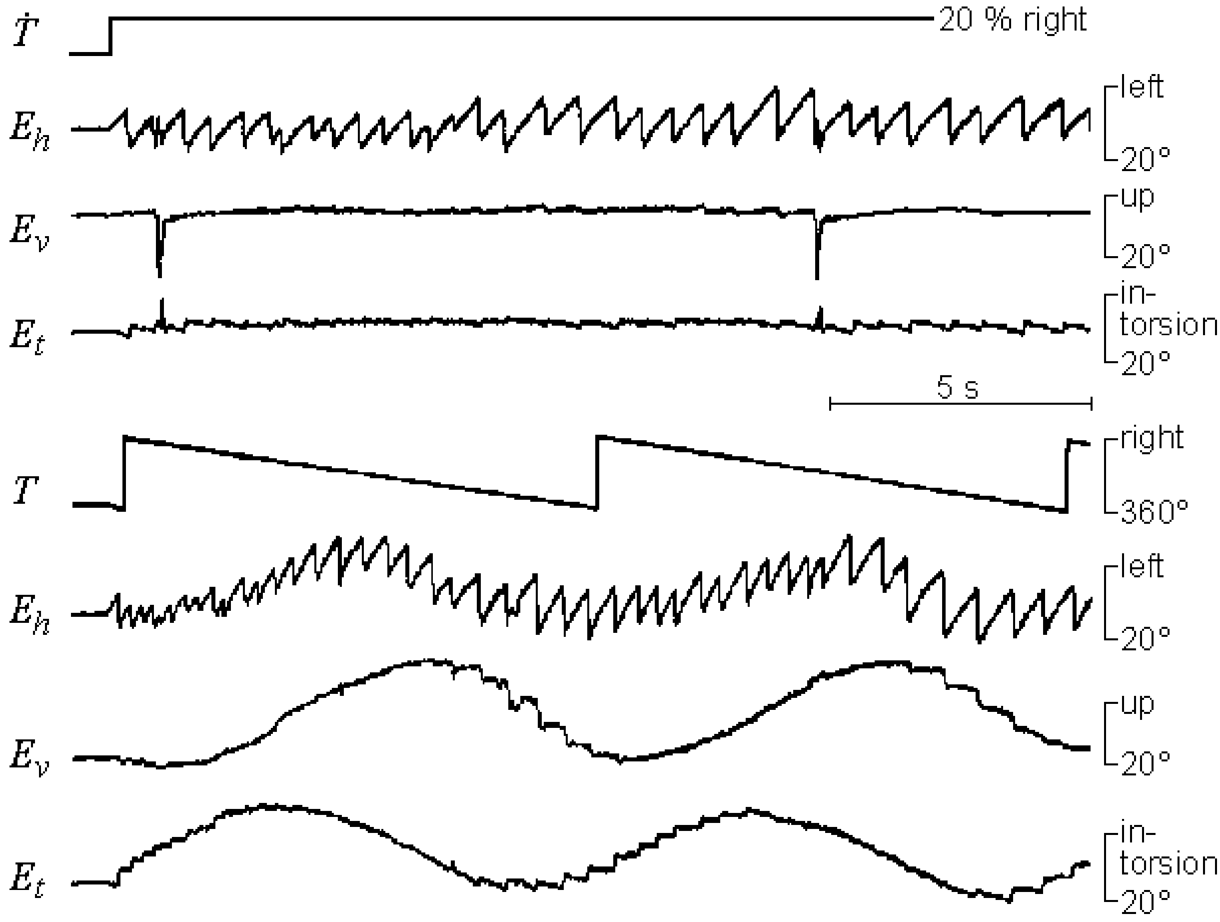


Figure 3. Horizontal, vertical, and torsional eye movements elicited in a rat by constant velocity rotation on a turntable. Upper panel: Eye movements elicited by constant-velocity rotation of the animal around an earth vertical axis relative to a stationary visual surround. The animal was standing on a horizontal platform with the head restrained in natural position (about 40° nose down). Slow horizontal tracking movements E_h, interrupted by fast repositioning movements, stabilize the eye in space (horizontal nystagmus) with a slow phase velocity (not shown) closely matching table velocity T. Vertical E_v, and torsional E_t eye position are not modulated. Note the blinks and the small saccadic shifts in the vertical and torsional component coupled with the horizontal quick phases. Lower panel: Horizontal, vertical, and torsional eye movements elicited by constant velocity rotation of the animal around an off-vertical axis in darkness. The platform with the animal was tilted by 30°. Horizontal eye position E_h shows a nystagmus with slow phase movements directed in an opposite direction from table rotation T. Vertical E_v and torsional E_t eye position exhibit a slow oscillation that is phase locked to table rotation. Slow phase velocity of horizontal eye movements (not shown) closely matches table velocity while vertical and torsional eye movements compensate in part for the continuously changing head orientation relative to the direction of gravity (gain about 0.4). Note the small saccadic shifts in vertical and torsional eye position coupled with quick phases of horizontal nystagmus. No correction for true torsion nor for deviation from space quadrature of the dual-search coil (σ = 96°).

Saccadic eye movements in the rat are often confined to the horizontal plane (E_h = output $A1$ of Fig. 2) showing only minor vertical (E_v = output $B1$ of Fig. 2) and torsional (E_t = output $B2$ of Fig. 2) components. In order to obtain the true ocular torsion, the output $B2$ must be corrected off line according to the above outlined procedure. In the illustrated case the direction coil (optic axis of the eye) was directed upwards by about 20°, resulting in an increased quantal noise of about 0.5° peak to peak in the horizontal trace E_h. The maximal useful time resolution of the recording is 5 ms. It is determined by the sampling rate of the Angle-Meter of 400 Hz.

When rotating an animal on a turntable about a vertical axis in front of a structured visual background, a horizontal nystagmus was elicited (upper panel of Fig. 3). Only minor vertical and torsional eye movement components were present. In contrast, horizontal, vertical, and torsional eye movements were elicited by rotating the animal about its vertical axis on a tilted platform (lower panel of Fig. 3). As mentioned above, the torsional movement components E_t have to be corrected off line by taking into account vertical eye position as well as a possible deviation from space quadrature of the dual search coil (see (7)). With the corrected torsion component at hand, the instantaneous orientation of the rotation axis of the eye in space can be calculated as outlined above.

Orthogonalization

In order to measure eye position in 3D space [1], two search coils, arranged in nonparallel planes, must be used. We shall assume that one coil with area vector $\underline{d}$ is attached in the frontal plane (direction coil) and another one with coil vector $\underline{t}$ (torsion coil) in a nonfrontal plane of the eye (or head). For simplicity, we shall also assume that the direction coil vector points along the line of sight and that the vectors $\underline{d}$, $\underline{t}$ and $\underline{d} \wedge \underline{t}$ establish a right-handed reference system. To calculate the torsion around the line of sight, we first have to determine the orthogonal complement of the vector $\underline{t}$ with respect to the vector $\underline{d}$:

$$\begin{aligned} \underline{t}^{\perp} &= (\underline{t} - (\underline{t} \bullet \underline{d})\, \underline{d}) \,/\, \| \underline{t} - (\underline{t} \bullet \underline{d})\, \underline{d} \| \\ &= (\underline{t} - \cos\sigma \bullet \underline{d}) \,/\, \sin\sigma \end{aligned} \tag{5}$$

Given the output voltages $A1$, $B1$, $A2$ and $B2$, the angle σ between the two coils is obtained with the help of (2). As to the torsion angle of the search coil with respect to the orientation of the direction coil, we refer to the plane determined by the axes e_x and e_y of the reference frame of the system. For a given position of the search coil, we define a reference torsion vector

$$\underline{t}_{ref} := (-\sin\alpha_1, \cos\alpha_1, 0) \tag{6}$$

such that $\underline{d} \bullet \underline{t}_{ref} = 0$. The torsion angle γ with respect to the reference torsion vector is given by

$$\begin{aligned} \cos\gamma &= \underline{t}^{\perp} \bullet \underline{t}_{ref} \\ &= \cos\beta_2 \sin(\alpha_2 - \alpha_1) / \sin\sigma \end{aligned} \tag{7}$$

where $\underline{t}^{\perp}$ is the orthogonal complement of the torsion vector $\underline{t}$. This relation defines the magnitude of the torsion angle γ. The sign of the torsion is obtained from the cross vector product $\underline{p} = \underline{t}_{ref} \wedge \underline{t}^{\perp}$. A torsion is positive if the first component of the vector $\underline{p}$ is pointing along $\underline{d} - d_3\underline{e}_3$. This means that

$$\gamma > 0 \text{ if } \cos\alpha_1 (\sin\beta_2 + \cos\sigma \sin\beta_1) / \sin\sigma > 0 \tag{8}$$

Assuming $\alpha_1 < \pi/2$ (and $\sin\sigma < \pi$), we finally have

$$\gamma > 0 \text{ if } \sin\beta_2 > -\cos\sigma \sin\beta_1 \tag{8'}$$

[1] H. Kasper and B. J. M. Hess, "Magnetic search coil ..." IEEE Trans. Biomed. Eng., vol. 38, no. 5, pp. 466-475, 1991